REVIEW SHEETS
ELEMENTARY ALGEBRA
MATH 65

A Summary of Concepts Needed to be Successful in Mathematics

The following sheets list the key concepts which are taught in the specified math course. The sheets present concepts in the order they are taught and give examples of their use.

WHY THESE SHEETS ARE USEFUL –

- To help refresh your memory on old math skills you may have forgotten.

- To prepare for math placement test.

- To help you decide which math course is best for you.

HOW TO USE THESE SHEETS –

- Students who successfully review spend from four to five hours on this material. We recommend that you cover up the solutions to the examples and try working the problems one by one. Then, check your work by looking at the solution steps and the answer. **Since you will not be allowed to use a calculator on the placement test, we recommend you do these review problems without a calculator.**

KEEP IN MIND –

- These sheets are not intended to be a short course. You should use them to simply help you determine at what skill level in math you should begin study. For many people, the key to success and enjoyment of learning math is in getting started at the right place. You will, most likely, be more satisfied and comfortable if you start onto the path of math and science by selecting the appropriate beginning stepping stone.

A. Use the slope-intercept form or the point-slope form to write the equation of a line when given a graph or other information.

1) Write the equation of:

    a) the line shown in the graph (grid intervals represent one unit);

    b) the line with slope $-1/2$ that passes through the point $(-5, 6)$;

    c) the line parallel to $3x + y = 4$ passing through the point $(0, 5)$;

    d) the line passing through the points $(-3, 6)$ and $(1, -4)$.

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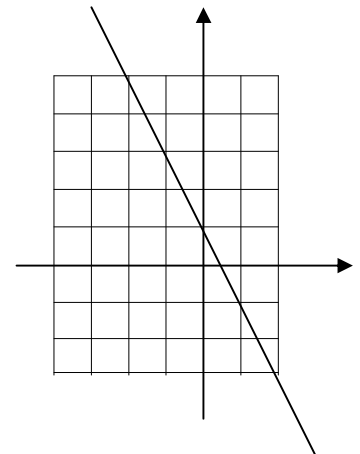
B. Plot a set of data, draw a line to fit the data, and develop a linear model of the data. Use the model to answer questions related to the data.

2) The following table shows the cost of a first class stamp in the given year. Use this information to answer the following questions.

Year	1981	1985	1988	1991	1995	1999	2001
Cost of Stamp (¢)	20	22	25	29	32	33	34

    a) On a sheet of graph paper, draw a scatter plot of these data. Let x = number of years since 1980 and let y = the cost of a stamp, in cents.

    b) On your scatter plot, draw a line to fit these data.

    c) Find a linear model for this data by writing the equation of your line.

    d) What is the slope of your line, and what is its meaning in this application?

    e) Use your equation to predict the cost of a stamp in the year 2010.

    f) Use your equation to find the x- and the y-intercepts of your graph, and tell what each one means in this application.

C. Solve linear systems of two equations algebraically and graphically.

3) Solve each system of linear equations by graphing.

    a) $\begin{aligned} y &= 2x + 4 \\ y &= -3x - 6 \end{aligned}$
    b) $\begin{aligned} y &= -\frac{1}{2}x + 4 \\ x + 2y &= 6 \end{aligned}$

4) Solve each system of linear equations using the given method.

    a) $\begin{aligned} 5x - 2y &= -7 \\ y - 3x &= 5 \end{aligned}$ by substitution
    b) $\begin{aligned} 5x + 4y &= 10 \\ -3x - 5y &= 7 \end{aligned}$ by addition

5) There are three possibilities for the number of solutions that a system of two linear equations may have. What are the three possibilities and what name is given to each?

D. Simplify expressions using the rules of exponents.

6) Simplify each of the following expressions and write the results without negative exponents.

    a) $a^4 \cdot a^6$
    b) $\left(a^4\right)^6$
    c) $\dfrac{a^4}{a^6}$
    d) $\dfrac{a^6}{a^4}$

    e) $\left(2x^2\right)^3$
    f) $3x^{-2}$
    g) $(3x)^{-2}$
    h) $4y^0$

i) $(4y)^0$
j) $\left(\dfrac{-3u}{v^2}\right)^4$
k) $\left(\dfrac{5}{z}\right)^{-3}$
l) $\dfrac{16c^{-4}}{-8c^8}$

m) $x^{-2}+3^{-2}$
n) $(x+4)^{-2}$
o) $\dfrac{6x^{-3}(2x^{-5})}{-3x^{-7}}$

E. Use scientific notation.

7) Write each number in scientific notation.

   a) $586,000$
   b) $0.000\,0089$
   c) 483×10^{-5}

8) Use scientific notation to compute each answer.

   a) $(48,000,000)(380,000,000)$
   b) $\dfrac{0.000\,000\,005}{0.0002}$

F. Use the terminology of polynomials and add, subtract, multiply and divide polynomials.

9) Write an example of each type of polynomial described below:

   a) A trinomial of degree two.
   b) A monomial of degree 5.

10) For each of the following expressions, tell whether or not it is a polynomial. If it is a polynomial, tell what the degree is.

   a) $3-4x+5x^2-2x^5$
   b) $\dfrac{1}{x}+5x-6$
   c) $\dfrac{a^4}{3}+a^2-6a^8$

11) Perform the following operations and simplify.

   a) $(x^2-6x+5)-(3x^2-5x+4)$
   b) $(6a^5-5a^3+2)+(3a^3-2a^2+7a)$

   c) $-3x(2x^2-6x+1)$
   d) $(-3m^4n^5)(-5mn^3)$

   e) $(x-4)(x-5)$
   f) $(2x+3)(3x-1)$

   g) $(y-4)(y^2-6y+2)$
   h) $\dfrac{4x^3-6x^2+10x}{-2x}$

12) Use "Special Products" to compute the following.

   a) $(2x+3)^2$
   b) $(3x^4-2)^2$

G. Factor polynomials by removing a common monomial factor, factor trinomials, and factor special products.

13) Factor each expression completely.

a) $30a^9 - 42a^4 + 54a^8$ b) $45x^2 - 18x - 27x^3$

c) $x^2 + 8x - 15$ d) $x^2 - 7x - 8$

e) $2x^2 + 11x + 12$ f) $x^3 - 3x^2 - 40x$

g) $8x^2 - 18x + 10$ h) $a^2 - 36$

i) $49 - w^2$ j) $3x^2 - 18x + 27$

k) $4y^2 - 1$ l) $9x^2 - 12x + 4$

H. Evaluate expressions involving square roots or cube roots.

14) Evaluate: a) $\sqrt{121}$ b) $\sqrt[3]{8}$

I. Use the properties of square roots to simplify expressions involving square roots and to perform operations with square roots.

15) Simplify each of the following roots, if possible. Assume all variables are positive numbers.

a) $\sqrt{x^6}$ b) $\sqrt{20}$ c) $\sqrt{a^3}$

d) $\sqrt{48x^5 y^{10}}$ e) $\dfrac{3}{\sqrt{16}}$ f) $\sqrt[3]{\dfrac{x^5}{125y^3}}$

g) $\sqrt{5} - 4\sqrt{5}$ h) $\sqrt{12} + 2\sqrt{27}$

J. Multiply and divide rational expressions.

16) a) $\dfrac{6x-12}{x^2-4} \cdot \dfrac{x+2}{3x}$ b) $\dfrac{4x^2+10}{x-3} \div \dfrac{6x^2+15}{x^2-9}$

K. Add and subtract rational expressions with the same denominator.

17) a) $\dfrac{3x+1}{4x-2} - \dfrac{x+1}{4x-2}$ b) $\dfrac{5x-2}{3x-4} + \dfrac{2x-3}{4-3x}$

L. Solve a quadratic equation by factoring or by extraction of roots.

18) Solve each quadratic equation.

    a) $x^2 = 100$ b) $2x^2 + 1 = 19$ c) $x^2 - 16 = 0$

    d) $(x+3)^2 = 81$ e) $(2x-3)^2 = 17$ f) $9x^2 + 27x = 0$

    g) $x^2 + 6x + 9 = 0$ h) $3x^2 = -21x - 18$ i) $2x^2 - 5 = 3x$

19) Solve the formula $V = \pi r^2 h$ for r.

M. Solve application problems including applications of linear systems and of the Pythagorean Theorem.

20) The perimeter of a rectangle is 50 yards, and its length is 9 yards greater than its width, w. Find the dimensions of the rectangle.

21) ABC Truck Rental charges $20 per day plus 50 cents per mile, while Murtz Truck Rental charges $35 per day plus 40 cents per mile. How far would you have to travel in one day for the cost from both rental companies to be the same?

22) If you drop a stone from a bridge 50 feet above the water, the height h of the stone t seconds after you drop it is given in feet by: $h = 50 - 16t^2$.

    a) What is the height of the stone after 1 second?

    b) When does the stone hit the water?

23) Find the unknown length in the right triangle shown below.
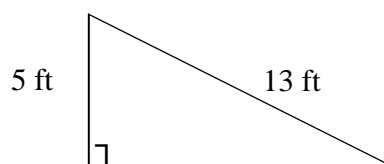
24) A guy wire is to be used to support an antenna that is 7 meters tall. The wire will be attached to the top of the antenna and to a point on the ground 24 meters from the base of the antenna. How long a wire is needed?

N. Recognize values of a variable that make an algebraic fraction undefined, and reduce algebraic fractions to lowest terms.

25) For what value(s) of x is each of the following algebraic fractions undefined?

a) $\dfrac{x-1}{x}$

b) $\dfrac{5}{2x-6}$

c) $\dfrac{x+4}{x^2-4}$

d) $\dfrac{x-3}{x^2+6x-16}$

26) Simplify by reducing to lowest terms.

a) $\dfrac{2ab}{6a^2b}$

b) $\dfrac{3ab+3ac}{5b^2+5bc}$

c) $\dfrac{x+4}{x^2-4}$

d) $\dfrac{x^2-1}{x^2+4x+3}$

e) $\dfrac{2-x}{x-2}$

f) $\dfrac{x^2+x-2}{2x+4}$

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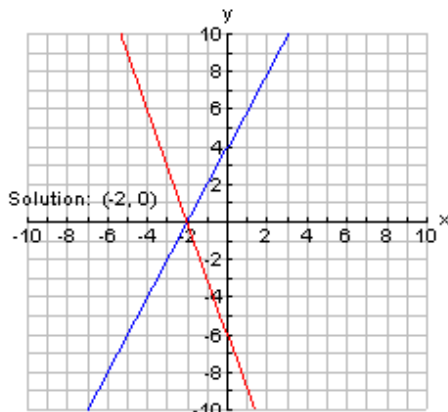# Answers for the Math 65 Review Objectives and Sample Problems
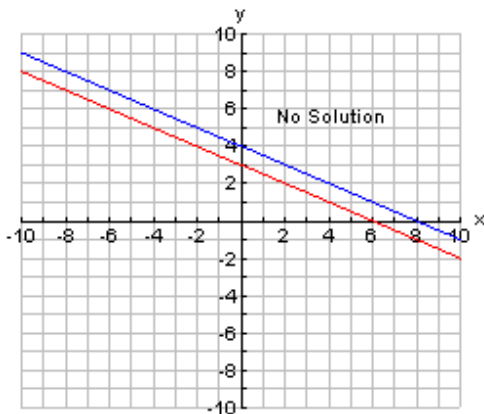
1) a) $y = -2x + 1$ b) $y = \dfrac{7-x}{2}$

   c) $y = -3x + 5$ d) $y = -\dfrac{5}{2}x - \dfrac{3}{2}$

2) Linear Modeling (answers will vary)

3) a)

Solution: (-2, 0)

3) b)

No Solution

4) a) (–3, –4)   b) (6, –5)

5) Possible number of solutions: zero (inconsistent), one (consistent), or infinitely many (dependent)

6) a) a^{10} b) a^{24} c) $\dfrac{1}{a^2}$

d) a^2 e) $8x^6$ f) $\dfrac{3}{x^2}$

g) $\dfrac{1}{9x^2}$ h) 4   i) 1

j) $\dfrac{81u^4}{v^8}$ k) $\dfrac{z^3}{125}$ l) $\dfrac{-2}{c^{12}}$

m) $\dfrac{1}{x^2} + \dfrac{1}{9}$ or $\dfrac{9+x^2}{9x^2}$

n) $\dfrac{1}{(x+4)^2}$ or $\dfrac{1}{x^2+8x+16}$ o) $-\dfrac{4}{x}$

7) a) 5.86×10^5 b) 8.9×10^{-6}

   c) 4.83×10^{-3}

8) a) 1.824×10^{16} b) 2.5×10^{-5}

9) a) One possible answer: $5x^2 + 7x - 1$

   b) One possible answer: $4x^5$

10) a) a polynomial of degree five

   b) not a polynomial

   c) a polynomial of degree eight

11) a) $-2x^2 - x + 1$

   b) $6a^5 - 2a^3 - 2a^2 + 7a + 2$

   c) $-6x^3 + 18x^2 - 3x$

   d) $15m^5 n^8$

   e) $x^2 - 9x + 20$

   f) $6x^2 + 7x - 3$

   g) $y^3 - 10y^2 + 26y - 8$

   h) $-2x^2 + 3x - 5$

12. a) $4x^2 + 12x + 9$
   b) $9x^8 - 12x^4 + 4$

13) a) $6a^4\left(5a^5 - 7 + 9a^4\right)$
   b) $-9x(3x-2)(x-1)$
   c) prime
   d) $(x-8)(x+1)$
   e) $(2x+3)(x+4)$
   f) $x(x-8)(x+5)$
   g) $2(4x-5)(x-1)$
   h) $(a+6)(a-6)$
   i) $(7-w)(7+w)$
   j) $3(x-3)^2$
   k) $(2y-1)(2y+1)$
   l) $(3x-2)^2$

14) a) 11   b) 2

15) a) x^3 b) $2\sqrt{5}$
   c) $a\sqrt{a}$ d) $4x^2 y^5 \sqrt{3x}$
   e) $\dfrac{3}{4}$ f) $\dfrac{x\sqrt[3]{x^2}}{5y}$
   g) $-3\sqrt{5}$ h) $8\sqrt{3}$

16) a) $\dfrac{2}{x}$ b) $\dfrac{2(x+3)}{3}$

17) a) $\dfrac{x}{2x-1}$ b) $\dfrac{3x+1}{3x-4}$

18) a) $x = \pm 10$ b) $x = \pm 3$

c) $x = \pm 4$ d) $x = 6, x = -12$

e) $x = \dfrac{3 \pm \sqrt{17}}{2}$ f) $x = -3, x = 0$

g) $x = -3$ h) $x = -6, x = -1$

i) $x = -1, \; x = \dfrac{5}{2}$

19) $r = \pm\sqrt{\dfrac{V}{\pi h}}$

20) <u>length:</u> 17 yards,   <u>width:</u> 8 yards

21) You would have to travel 150 miles in one day for the cost to be the same.

22) a) The height of the stone after 1 second is 34 feet.

   b) The stone hits the water after *approximately* 1.8 seconds

23) a) 12 ft   24) A 25 m guy wire is needed

25) a) $x = 0$ b) $x = 3$
   c) $x = \pm 2$ d) $x = -8, \; x = 2$

26) a) $\dfrac{1}{3a}$ b) $\dfrac{3a}{5b}$
   c) cannot be reduced
   d) $\dfrac{x-1}{x+3}$ e) -1
   f) $\dfrac{x-1}{2}$

4/24/13