REVIEW SHEETS
INTERMEDIATE ALGEBRA
MATH 95

A Summary of Concepts Needed to be Successful in Mathematics

The following sheets list the key concepts which are taught in the specified math course. The sheets present concepts in the order they are taught and give examples of their use.

WHY THESE SHEETS ARE USEFUL –

- To help refresh your memory on old math skills you may have forgotten.

- To prepare for math placement test.

- To help you decide which math course is best for you.

HOW TO USE THESE SHEETS –

- Students who successfully review spend from four to five hours on this material. We recommend that you cover up the solutions to the examples and try working the problems one by one. Then, check your work by looking at the solution steps and the answer.

KEEP IN MIND –

- These sheets are not intended to be a short course. You should use them to simply help you determine at what skill level in math you should begin study. For many people, the key to success and enjoyment of learning math is in getting started at the right place. You will, most likely, be more satisfied and comfortable if you start onto the path of math and science by selecting the appropriate beginning stepping stone.

1) *Solve linear equations.*

   Solve: $\dfrac{3z}{5} - \dfrac{z-3}{2} = \dfrac{z+2}{3}$

2) *Solve an equation or formula for a specified variable.*

   a) Solve for y: $x = \frac{2}{3}(y+4)$ 　　　　　b) Solve for x: $rx = 11x + 7$

3) *Use formulas and mathematical models*.

The mathematical model $V = C(1 - rt)$ is a relationship among the quantities C (original cost of a piece of equipment), r (percent steady depreciation rate expressed as a decimal), t (age, in years, of the equipment), and V (current value). What is the value after 5 years of a weaving loom that was originally purchased for $2100 and has steadily decreased in value at a rate of 6% per year?

4)   **Solve an application problem by setting up a linear equation.**

a)   (Percent)   The price of an item is increased by 40%. When the item does not sell, it is reduced by 40% of the increased price. If the final price is $220.00, what was the original price? What single percent reduction of the original price is equivalent to increasing by 40% then reducing by 40%?

b)   (Interest)   A real estate agent receives a commission of 8% of the selling price of a house. What should be the selling price so that the seller can get $87,400?

5)   **Find the area, perimeter, volume, and surface area of common shapes (rectangle, square, triangle, circle, rectangular solid).**
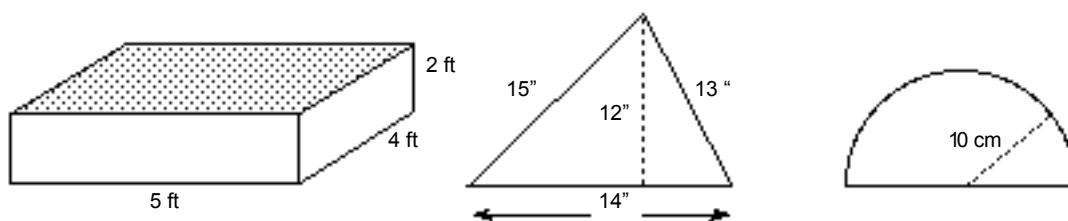
a) For the rectangular solid shown above left, find each of the following and write proper units.

   i) Perimeter of the top surface (shaded)

   ii) Area of the top surface (shaded)

   iii) Volume of the solid (box)

   iv) Total surface area including all six surfaces.

b) For the triangle above, find its area and perimeter.

c) For the semicircle above, find its area and perimeter.

6) **Find the x- and y-intercepts of a line from its equation and from a graph.**

    a) Find the coordinates of the *x*- and *y*-intercepts of the line A in the graph below.

    b) Find the *x*- and *y*-intercepts of the line whose equation is $3x - 2y = 6$.

    c) Sketch the line whose equation is $3x - 2y = 6$ by using its *x*- and *y*-intercepts.

7) **Determine the slope of a line by counting and the ratio rise/run, and by the slope formula.**

    a)   Determine the slope of a line that passes through the points $(0, 2)$ and $(5, 4)$.

    b)   Determine the slope of line A and line B shown on the graph at right.

    c)   Determine the slope of a horizontal line that passes through the point $(5, -20)$.
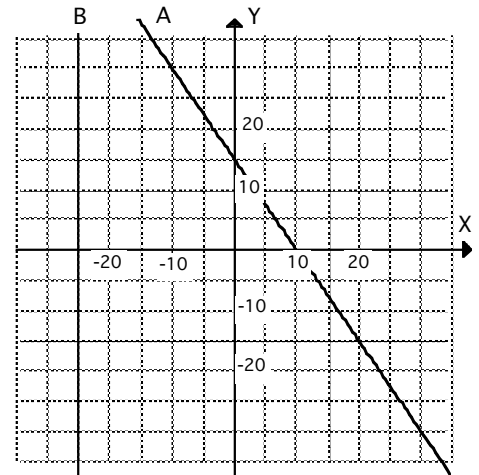
8) **Sketch a line having a specified slope.**

    Sketch an example of a line for each given slope:
        $-2/3$, 2, 0, undefined.

9) **Given a linear equation in two variables, convert it to slope-intercept form and determine the slope and y-intercept of the graph of the line.**

    Write $2x + 3y = 18$ in slope-intercept form, then find the slope and *y*-intercept of its line.

10) **Determine if two lines are parallel or perpendicular or neither.**

    a) Plot the four points (1, 4), (3,2), (4, 6), and (2, 8) and join them consecutively with straight lines segments. Use slopes to show that a pair of opposite sides of the figure are parallel and to show that a pair of adjacent sides are not perpendicular.

    b) Determine whether these two lines are parallel, perpendicular, or neither: $3x - 2y = 6$ and $2x + 3y = -6$.

11) **Given the graph of a vertical or horizontal line write its equation; given its equation sketch its graph.**

    a) What is the equation of the line B shown in the graph above?

    b) Sketch the graph of the line whose equation is $y = -2$.

12) **Write the equation of a specified line using the slope-intercept form and point-slope form.**

    a)   If a line passes through the points $(2, 5)$ and $(-3, 6)$, find an equation of the line in point-slope form and in slope-intercept form.

    b)   Find the equation of a line which passes through $(1, -3)$ and is perpendicular to the line $2x - y = 1$.

13) *Interpret information depicted in graphs.*

Suppose in a city riders pay a certain amount for each time they ride a bus. For frequent users a monthly pass is available for purchase. With a pass each ride costs 50 cents less than the regular price. The total monthly cost with and without a pass are represented by the two graphs at right. The horizontal axis represents the number of times a bus is used each month.
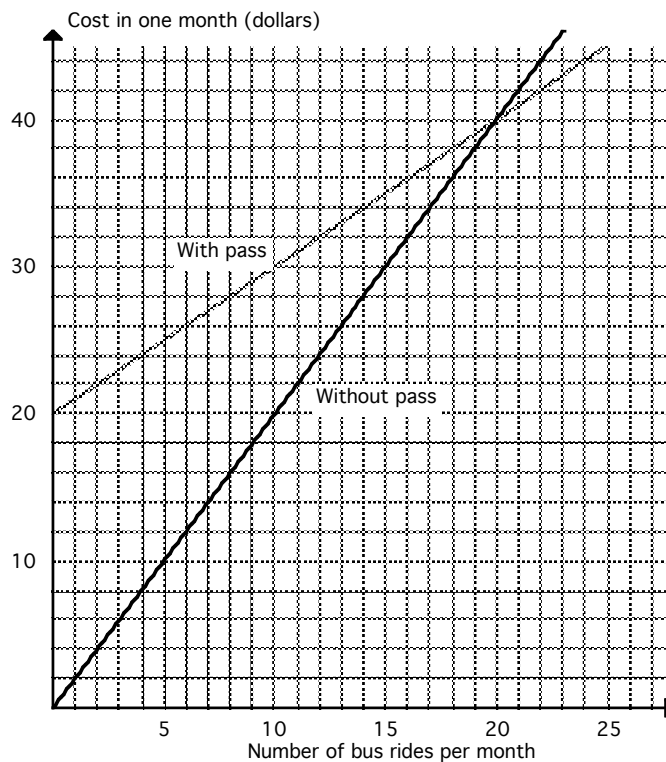
a) If a person anticipates using the bus 15 times in a month, should a pass be purchased?

b) For 15 rides in a month, what is the difference in total cost with and without a pass?

c) What is the *break-even* point, that is, for how many rides would the total cost be the same?

d) What is the cost of a monthly pass?

Cost in one month (dollars)

With pass

Without pass

Number of bus rides per month

14) *Interpret the practical meaning of a slope and intercepts in a linear application problem.*

a) Consider the line representing a rider with a monthly pass in the graph above.
   i) What is the value and units of the slope of the line?
   ii) What is the practical meaning of this slope?
   iii) What is the practical meaning of the vertical-axis intercept (y-intercept)?

b) The model $N = 34t + 1549$ describes the number of nurses N, in thousands, in the United States t years after 1985. Find the vertical-axis intercept (N-intercept) and the slope of this linear model. Explain in a clear sentence or two what each of these numbers means in practical terms.

15) *Decide whether an ordered pair is a solution of a linear system.*

Given the system consisting of $x - 3y = 2$ and $y = 6 - x$, is $(8, 2)$ a solution? Is $(2, 0)$? Is $(5, 1)$?

16) *Solve a linear system graphically.*

a) Solve the system consisting of the linear equations $y = (1/2)x + 3$ and $y = 7$ by graphing each equation.

b) Solve the following system graphically: $x - 3y = 2$ and $y = 6 - x$.

17) *Solve a linear system algebraically by the substitution method and the addition method.*

a) Solve the system by the substitution method. $4x + y = 5$ and $2x - 3y = 13$.

b) Solve the system by the addition method. $x - 2y = 5$ and $5x - y = -2$.

18) *Solve an application problem using a linear system of equations.*

a) (Interest) Suppose $20,000 was invested, part in a stock expected to pay 7.5% interest over 2 years and the rest in bonds paying 5.4% interest for 2 years. If the total interest earned in 2 years was $2496, how much was invested in the stock and in the bonds? Select and define variables, set up a system of equations, and solve.

4

b)   (Mixture) One container has an acid solution containing 30% acid and a second contains 60% acid. If parts of the two are to be mixed together in order to create 60 gallons of a solution which is 50% acid, how many gallons from each container should be used?

19)   *Use integer exponents including those which are positive, negative, or zero.*

Simplify.   a) $(-3x^2y^5)^2$ b) $(5x^3y)(4x^{-2}y^3)^{-2}(2xy)^0$ c) $\dfrac{25a^{-8}b^2}{75a^3b^{-4}}$

20)   *Use scientific notation*. Be able to do by hand and with a calculator.

Simplify by hand and express the result using scientific notation. $\dfrac{(3600)(0.03)}{(120{,}000)(0.00090)}$

21)   *Recognize the words monomial, binomial, and trinomial, degree, and leading coefficient when describing a polynomial.*

a)   Write the polynomial $x^2 - 4x^3 + 9x - 12x^4 - 6$ in descending powers of x. Find the degree of the polynomial and the value of the leading coefficient.

b)   Write an example of a trinomial whose degree is 6 with a leading coefficient of 5. There are many possibilities.

22)   *Add and subtract polynomials.*

a) Add $5x^3 - 9x^2 - 8x + 11$ and $17x^3 - 5x^2 + 4x - 3$.

b) Subtract $5x^3 - 9x^2 - 8x + 11$ from $17x^3 - 5x^2 + 4x - 3$.

23)   *Multiply polynomials.*

Find each product.

a) $-2x^2(5x^3 - 8x^2 + 7x - 3)$ b) $(x + 5)(x^2 - 5x + 25)$

c) $(5x + 3)(7x - 1)$ d) $(2x - 3)(2x + 3)$

24)   *Divide a polynomial by a monomial.*

Simplify. a) $\dfrac{x + 4}{x}$ b) $\dfrac{6x^7 - 3x^4 + x^2}{3x^3}$

25)   *Factor out the greatest common factor (GCF) of a polynomial*.

Factor out the GCF in each expression.

a) $2a^2b - 5ab^2 + 7a^2b^2$ b) $7(x - 3) + y(x - 3)$ c) $-3x^2 + 27x$

26)   *Factor by the grouping method.*

a) Factor the polynomial $x^3 - 3x^2 + 4x - 12$ by the grouping method.

b) Factor the trinomial $12x^2 - 5x - 2$ by the grouping method (also known as the a-c method).

27)   *Factor quadratic trinomials completely.*

Factor: a) $6x^2 - x - 35$ b) $15w^3 - 25w^2 + 10w$

28) **Factor a polynomial that is a difference of squares.**

Factor $36x^2 - 49y^2$

29) **Factor a polynomial that is a difference of cubes or a sum of cubes.**

a) Factor $x^3 - 8y^3$ b) Factor $125b^3 + 64$

30) **Solve a quadratic equation by factoring and applying the zero-factor property.**

Solve each of the following quadratic equations.

a) $6x^2 - x - 35 = 0$ b) $25 = 30x - 9x^2$

c) $3x^2 = 12x$

31) **Recognize and use vocabulary associated with quadratic equations.**

Given the quadratic equation $3x^2 = x - 5$. Fill in the blanks with the appropriate responses.

a) In *standard form* the equation is written as _____.

b) The *degree* of the quadratic expression is _____. The *leading coefficient* is _____.

c) The *constant term* of the quadratic expression is _____.

d) The *coefficients a, b,* and *c* are, respectively, _____ , _____ , and _____ .

32) **Determine the values of the variable for which a rational expression is undefined.**

The expression $\dfrac{3}{x^2 - 1}$ is not defined for what value(s) of x?

33) **Simplify a rational expression by reducing to lowest terms.**

Simplify.   a) $\dfrac{3a + 9}{a^2 + 6a + 9}$ b) $\dfrac{4 - x}{2x - 8}$

34) **Multiply and divide rational expressions.**

a) Perform the operation indicated and simplify, if possible. $\dfrac{5x^2 - 5}{3x + 12} \cdot \dfrac{x + 4}{x - 1}$

b) Perform the operation indicated. $\dfrac{x^2 - y^2}{(x + y)^2} \div \dfrac{x - y}{4x + 4y}$

35) **Add and subtract rational expressions.**

Perform the operation indicated.

a) $\dfrac{x + 5y}{x + y} + \dfrac{x - 3y}{x + y}$ b) $\dfrac{5}{2t - 8} - \dfrac{3}{t - 4}$ c) $\dfrac{1}{x} + \dfrac{2}{x - 5}$

36) **Simplify a complex fraction.**

Simplify: $\dfrac{1+\dfrac{4}{x}}{x^2}$

37) **Solve an equation containing algebraic fractions.**

Solve each equation. Remember to check for extraneous solutions.

a) $\dfrac{1}{x}-\dfrac{2}{3}=\dfrac{5}{6x}$

b) $\dfrac{3}{x^2-9}+\dfrac{4}{x+3}=1$

38) **Solve an application problem involving a rational expression.**

When a manufacturer produces x number of its product, the average cost in dollars per unit is modeled by $C=\dfrac{5x+5000}{x}$. Determine the number of units it must produce in order for the average cost of the item to be $9.75. According to this model, can the average cost be made to drop to $4.00 per item?

39) **Use function notation to evaluate a function.**

a) If $h(x)=2x^2+3x-1$, find $h(0)$, $h(-4)$, $h(c)$, and $h(5r)$.

b) Given the temperature function $g(F)=(5/9)(F-32)$, complete the input-output table below.

F	-13	4	32	212	451
$g(F)$					

40) **Write an interval of numbers using inequality notation, interval notation, and graph on a number line.**

a) If $-1\le x<6$, write this interval using interval notation, and graph it on a number line.

b) Let the value of the variable x be at most 10. Represent the values of x using inequality notation and interval notation, then sketch on a number line.

41) **Solve a linear inequality and graph the solution on a number line.**

a) Solve $-4x+6\ge 10$ and sketch the solution on a number line.

b) Solve $7(2x-1)<9x+11$ and express the solution using interval notation.

42) **Simplify a radical expression containing perfect powers of numbers without using a calculator.**

Simplify the following. a) $\sqrt[4]{16}$ b) $\sqrt[3]{-\frac{1}{64}}$

43) **Use a scientific calculator for evaluating radicals and rational exponents.**

Use your calculator to compute each of the following. a) $\sqrt[4]{500}$ b) $19^{6/5}$

44) **Express a radical using rational exponents, and vice versa.**

a) Rewrite $\sqrt[5]{L^2 m^4}$ using rational exponents.   b) Express $(8x)^{2/3}$ using a radical.

45) ***Simplify an expression containing rational exponents.***

a) Evaluate the following without using a calculator.   i) $36^{1/2}$ ii) $5^{6/3}$ iii) $9^{3/2}$

b) Simplify and express answer using a rational exponent. $(7y^{1/3})(2y^{1/4})$

46) ***Use the product rule for radicals; simplify square root radicals.***

Simplify   a) $\sqrt{2xy^3}\sqrt{22x^3y^2}$ b) $\sqrt{32x^{13}}$

47) ***Use the quotient rule for radicals; use the quotient rule to simplify an expression.***

Simplify   a) $\dfrac{\sqrt{54x^3}}{\sqrt{6x}}$ b) $\sqrt{\dfrac{2x^5}{9}}$

48) ***Rationalize a denominator of one term involving a square root.***

Rationalize the denominator in each expression.   a) $\dfrac{1}{\sqrt{2}}$ b) $\sqrt{\dfrac{5}{6}}$

49) ***Add and subtract like radicals.***

Simplify $4\sqrt{12} - 2\sqrt{75}$

50) ***Apply the distributive property with an expression containing radicals.***

Simplify.   a) $2\sqrt{3}(5 + \sqrt{2})$ b) $(\sqrt{2} + \sqrt{7})(\sqrt{3} + \sqrt{5})$

51) ***Solve an equation containing radicals.***

Solve the equation. (Check for extraneous solutions.)

a) $2 + \sqrt{3x - 2} = 7$ b) $x = \sqrt{6x + 7} - 2$ c) $\sqrt[3]{x^2 + 6x} + 2 = 0$

52) ***Solve an application problem involving radicals.***

The formula $s = \sqrt{30fd}$ models the relationship between the distance d, in feet, that a car skids after the brakes are applied, the condition of the pavement f, a number, and the speed of the car s, in miles per hour, just before the brakes were applied. Suppose the length of a skid is 47.5 feet and the road was slightly damp so that f is taken to be 0.63. What was the speed of the vehicle according to the model?

53) ***Write the square root of a negative number in terms of i, the imaginary unit.***

Express $\sqrt{-45}$ in terms of i.

54) ***Solve a quadratic equation by the square root method (extracting square roots).***

Solve   a) $3x^2 - 24 = 0$ b) $(3y - 1)^2 = 16$

55) ***Write imaginary solutions to quadratic equations using complex numbers.***

Solve   a) $x^2 + 16 = 0$ b) $(2x - 3)^2 = -4$

56) **Solve a quadratic equation by completing the square.**

Solve by completing the square. Leave your answers in exact form. $x^2 + 6x - 5 = 0$

57) **Find the distance between two points of a rectangular coordinate system by applying the Pythagorean Theorem or the Distance Formula.**

Determine the distance between the points $(4, 3)$ and $(-6, 2)$.

58) **Find the discriminant of a quadratic equation and predict the number and type of solutions.**

Compute the discriminant, and then predict the number and type of solutions the equation will have.

    a) $-2x^2 + 6x = -1$ b) $4y^2 + 2y + 5 = 0$

59) **Solve a quadratic equation using the quadratic formula.**

Solve each equation using the quadratic formula. Leave your answers in exact form simplified.

    a) $-2x^2 + 6x = -1$ b) $t^2 = -6t - 13$

60) **Solve a variety of equations that lead to a quadratic equation.**

Solve each equation by the algebraic method of your choice.

    a) $(x - 10)(x - 2) = -7$ b) $\dfrac{5}{x+1} + \dfrac{x-1}{4} = 2$

61) **Solve an application problem requiring setting up a quadratic equation.**

    a)   A rectangular garden has a path 50 feet long along the diagonal. The length of the garden is twice as long as its width. Find the dimensions of the garden exactly and also approximately rounded to the nearest tenth.

    b)   The height of a sign in the shape of a triangle is 4 inches less than its base. Find the base and height of the sign if its area is 100 square inches. Round off your answer to the nearest tenth of an inch.

62) **Graph a quadratic function by constructing a table of values and plotting points.**

Given $f(x) = x^2 - 6x$, make a table of values selecting x values on both sides of $x = 3$, and construct graph.

63) **Solve a quadratic equation graphically.**

The graph of the quadratic function $y = x^2 - 2x - 5$ is shown below. Use the graph to estimate the solutions of the equation $x^2 - 2x - 5 = 0$ to one decimal point.

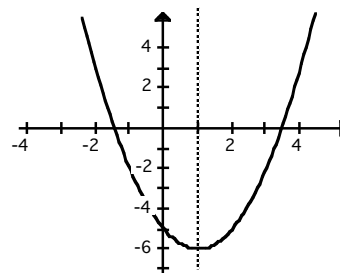64) **Use vocabulary associated with the graph of a quadratic function.**

Refer to the graph at right. Fill in the blanks.

    a) The shape of the graph is called a _____ .

    b) The line whose equation is $x = 1$ is called the axis (or line) of _____.

    c) The coordinates of the vertex of this graph are _____ .

    d) The vertex point $(1, -6)$ is also described as the _____ (minimum or maximum?) point of the parabola.

65) **Identify the leading coefficient of a quadratic function and determine whether its parabolic graph opens upward or downward.**

Determine the orientation of each parabola.   a) $y = 3x^2 - 18x - 5$ b) $d = 80 + 64t - 16t^2$

66) **Find the intercepts of the graph of a parabola algebraically.**

Find the x- and y-intercepts of the parabola given by the equation $y = 2x^2 + x - 5$. Write each intercept in coordinate form. Use calculator to round off answers to the nearest two decimals.

67) **Find the equation of the axis of symmetry and the coordinates of the vertex of a parabola.**

Find the equation of the axis of symmetry of the parabola and the coordinates of the vertex of the parabola given by the equation $f(x) = x^2 - 2x - 8$.

68) **Use the intercepts, the vertex, and symmetrical points as an aid in sketching the graph of a parabola.**

a)   Sketch the graph of the parabola whose equation is $y = x^2 - 2x - 8$ using its intercepts, its vertex, its axis of symmetry, and any additional points, if helpful.

b)   Sketch the graph of $y = 3x^2 - 8x + 4$.

69) **Find the maximum or minimum point on a parabola algebraically.**

Determine whether the parabola given by $y = 3x^2 - 18x - 5$ has a maximum or minimum point. Then find the coordinates of that point.

70) **Find the vertex and intercepts of a parabola and interpret their meanings in an application.**

A toy rocket is launched from a platform and rises straight up. The height h, in feet, of the rocket at any time t, in seconds, after launch is modeled by the formula $h = -16t^2 + 80t + 5$.

a) How high did the rocket travel?

b) How many seconds did it take to reach the highest point?

c) How many seconds after launch does the rocket hit the ground?

d) Without actually graphing, what is the vertical axis-intercept? What is the practical meaning of this intercept?

71) **Evaluate an exponential expression or function.**

If $f(x) = 5(2^x)$, find $f(-1)$ and $f(3)$.

72) **Solve several types of application problems involving exponential functions.**

a)   (Periodic compound interest) Suppose you have $6000 to invest for 5 years. Compare investing the amount in an account giving 5.6% compounded semiannually (two times per year) with an account giving 5.3% monthly. Which is better and how much more of a return do you get?

b)   (Continuous compounding) If $6000 is invested for 5 years in a program offering 5.3% compounded continuously, how much interest is earned?

c)   (Changing population) The increasing population of a town can be modeled by the function $P(t) = 1200(1.016)^t$ where t is number of years since 1970. What was the population in 1970? According to this model, what is the predicted population for 2010?

73) ***Change an equation in exponential form to logarithmic form, and vice versa.***

    a) Write the statement $\log_2 64 = 6$ in exponential form.

    b) Change $\log A = B$ to exponential form.

    c) Express the equation $\ln x = 3.5$ in exponential form.

    d) Write each of the following in logarithmic form: $3^4 = 81$ and $100 = e^x$.

    e) Fill in the blank: If $5^3 = 125$, it means that $\log_5 125 =$ _____ .

    f) If $1.7 = \log_2 T$, 2 must be raised to what exponent to get T?

74) ***Evaluate a logarithm using a scientific calculator.***

Find each of the following to 4 decimal places. If an answer does not exist, write "Does not exist"

       $\log 2.25$ $\log 225$ $\ln 7.06$ $\ln 706$ $\log (-40)$